

Embedding Virtual Manipulatives into Middle School Mathematics Curriculum

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KEYWORDS Attitude Toward Mathematics. *Mathematica*. Mathematics Achievement. *Matletik*

ABSTRACT The use of virtual manipulatives in mathematics education has increased in the last decade. Many researchers attempted to investigate the effects of using virtual manipulatives on mathematics education. The purpose of this study is to investigate the effects of extensive and intensive use of virtual manipulatives on the mathematics achievement and the attitudes of 8th grade students. For this purpose, the researchers used a dynamic software called *Matletik* written in *Mathematica* language and specially prepared for every grade of K-12 mathematics curriculum of Turkish National Education Ministry. The study was conducted using six classes of 8th grade students, selected from three different schools as a sample. Two classes of each school were randomly assigned as the treatment and the control groups. A pre and posttest experimental design was used to measure the efficiency of the method. In the treatment groups the software was used for 14 weeks, while the traditional instruction was given to the control groups. The results showed that mathematical achievement and the attitude toward mathematics could be improved upon through the use of virtual manipulatives.

INTRODUCTION

Virtual manipulatives are interactive, visual representations of dynamic objects that present opportunities for constructing mathematical knowledge (Moyer et al. 2002). Virtual manipulatives can help the students have a better understanding of mathematical concepts. Virtual manipulatives are usually in the form of Java or Flash applets, and they can be manipulated with widgets in a GUI (Graphical User Interface), such as a slider, a checkbox or a button. They are also defined as “computer based renditions of common mathematics manipulatives and tools” (Dorward 2002). Virtual manipulatives help develop students’ visualization skills by connecting words, pictures and symbols simultaneously. This simultaneous presentation can assist students in developing a solid understanding of mathematical concepts (Paivio 2007). With virtual manipulatives, it is possible to simplify the tasks, which are difficult or impossible to do on the whiteboard, generate examples or data, give students examples with which they can reinforce their knowledge on mathematics’ subjects. Vir-

tual manipulatives can also be used to present pictorial proofs, puzzles, animations and simulations. The use of physical and virtual manipulatives has recently increased in mathematics education. See Uribe-Flórez and Wilkins (2010) for a detailed literature review on manipulatives in mathematics education, and Hansen et al. (2016) for a recent study of virtual manipulatives on fractions. See also, Moyer-Packenham and Westenskow (2012) for a meta-analysis focused on the effects of virtual manipulatives on students’ achievement.

Objectives

The purpose of this study is to investigate the effects of using virtual manipulatives on mathematics achievement and attitudes of 8th grade students. An experimental research model with pre and post-test was considered as appropriate to examine these effects.

Math Achievement and Attitudes Toward Mathematics

An attitude is “a relatively enduring organization of beliefs, feelings, and behavioral tendencies towards socially significant objects, groups, events or symbols” (Hogg and Vaughan 2005). Attitudes may affect an individual’s behavior in the presence of attitude-objects in predictable ways (Ajzen and Fishbein 1980). The attitude toward mathematics is defined as “a lik-

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ing or disliking of mathematics, a tendency to engage in or avoid mathematical activity, a belief that one is good or bad at mathematics, and a belief that mathematics is useful or useless” (Neale 1969). Based on the experiences with mathematics at school, students may create general attitudes about its nature and value and about their own abilities and interest in doing it (Boekaerts and Simons 2003). Students’ attitudes are developed over a considerably long period of time and have powerful impacts on their effective engagement, participation and achievement in mathematics (Majeed et al. 2013). It is also a well-known fact that attitude plays a crucial role in learning (Neale 1969), and influences success and persistence in mathematics (Thorndike-Christ 1991). From research, it can be seen that attitudes toward mathematics have considerable effects on achievement (Dwyer 1993). Many instruments exist for measuring the students’ attitudes toward mathematics. Aiken’s Mathematics Attitudes Scales (MAS) (Aiken 1970) and the Fennema-Sherman Mathematics Attitudes Scales (FSMAS) (Fennema and Sherman 1976) are the most widely used ones. Both of them are the basis of the newly created math attitude scales. The Mathematics and Technology Attitudes Scale (MTAS) is generally used for investigating the students’ attitudes in learning mathematics with technology, measuring mathematics confidence, confidence with technology, attitude to learning mathematics with technology and two aspects (affective and behavioral) of engagement in learning mathematics (Pierce et al. 2007). The Attitudes Toward Mathematics Inventory (ATMI) (Tapia and Marsh 2004) is one of the latest scales being used, which contains 40 items and identifies four dimensions as self-confidence, value, enjoyment and motivation, along which attitudes toward mathematics could be measured. In this research the researchers have chosen ATMI instead of MTAS, because MTAS is appropriate for researches in which students use technological tools directly such as computer software, calculators, graphics calculators, computer algebra systems, spread sheets or statistics programs. Whereas, for virtual manipulatives used in the study, the user interacts with easy to use widgets such as sliders, buttons or check boxes only, and they are used in classrooms by teachers generally. In this study, the researchers examined the effects of an extensive use of virtual

manipulatives on attitudes toward mathematics and achievement in mathematics.

Mathematics and Anthropology

At first sight, though mathematics and anthropology would seem to have little in common, many researchers attempted to combine the two disciplines. Bishop (1988), for instance, states that anthropology is a useful tool for understanding the transmission of mathematical knowledge in society’s culture. He claims that placing mathematics in a cultural context, rather than approaching it as a discrete, isolated entity removed from the real world may have far-reaching implications for mathematics education (see, Connors 1990 for a detailed discussion on Bishop’s arguments). Some empirical studies (see for instance, Mesquita et al. 2013), show that the low achievement in mathematics may be caused by socio-cultural influences, which emanate from homes, peers, communities, and cultures. Pinxten and François (2013) introduced the concept of ‘multimathemacy’ as an alternative to the monolithic approach to mathematics, which is an educational perspective that invites the teaching of different cultural insights on counting, proportional thinking, mapping or spatial organization in preschool and bridges between academic mathematics and cultural knowledge traditions for schooling. The researchers of this paper believe that using virtual manipulatives can positively affect achievement in mathematics for students at all levels and of all abilities. That is true for almost every topic covered in elementary and middle school mathematics curricula. Virtual manipulatives as supplementary tools have the potential to be used in alternative and innovative educational systems and paradigms.

Another point the researchers want to make is that virtual manipulatives may offer relatively equal opportunities for students from low socioeconomic status backgrounds since many Internet sources provide them for free (see for instance, <http://nlvm.usu.edu/en/nav/vlibrary.html>).

Matletik

Matletik (www.matletik.com) is a dynamic software written in the *Mathematica* language, specialized for every grade of K-12 mathematics curriculum of Turkish National Education Ministry. It is used along with the free *Wolfram CDF Player* (<https://www.wolfram.com/cdf-player/>),

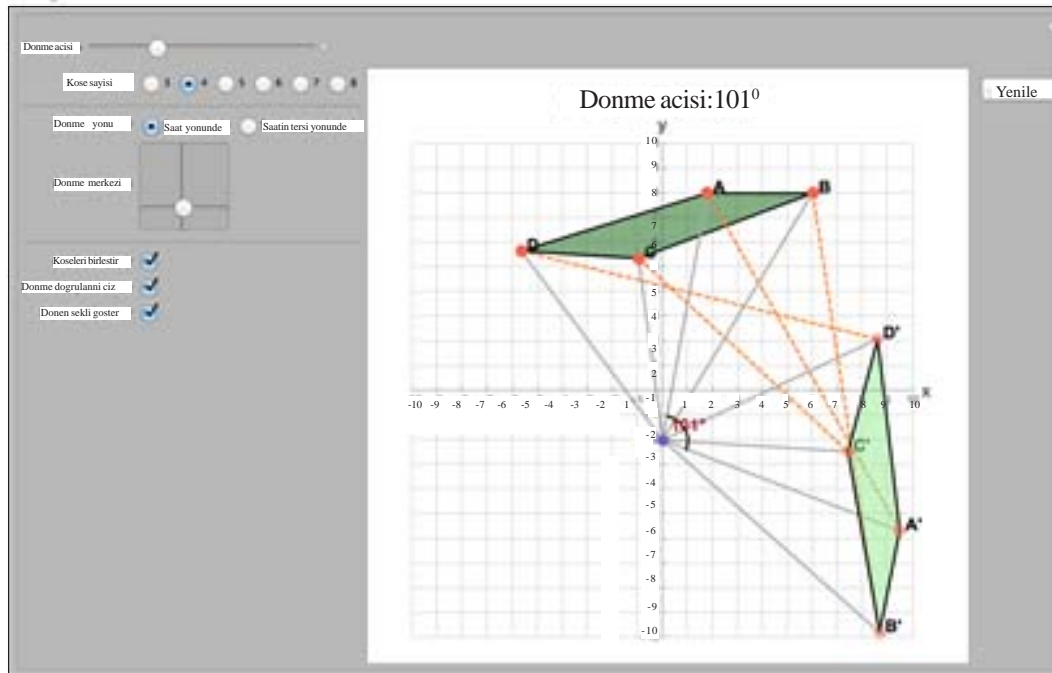
which is the platform for running interactive applications based on *Mathematica*. The virtual manipulatives of *Matletik* can be run offline on an interactive whiteboard or as a web-based application through a web browser. For this study, the researchers used 96 virtual manipulatives of *Matletik* for the 8th grade, having a range of characteristics such as pictorial proofs, animations, simulations, concept tutorials and tests in several different forms. Figure 1 shows some example snapshots of them. In (a), one can rotate an object of which the shape can be changed by dragging and adding the locators. The center, angle and direction of the rotation can also be changed. In (b), one can perform a die throwing experiment for a large number of times, and in (c) using the sliders of radius and height, one can watch the changes in the shape of a cone, its net and area as well. In (d), one can generate multiple-choice tests for the equation of line, and finally in (e), one can explore the geometric meaning of the product of two algebraic terms.

METHODOLOGY

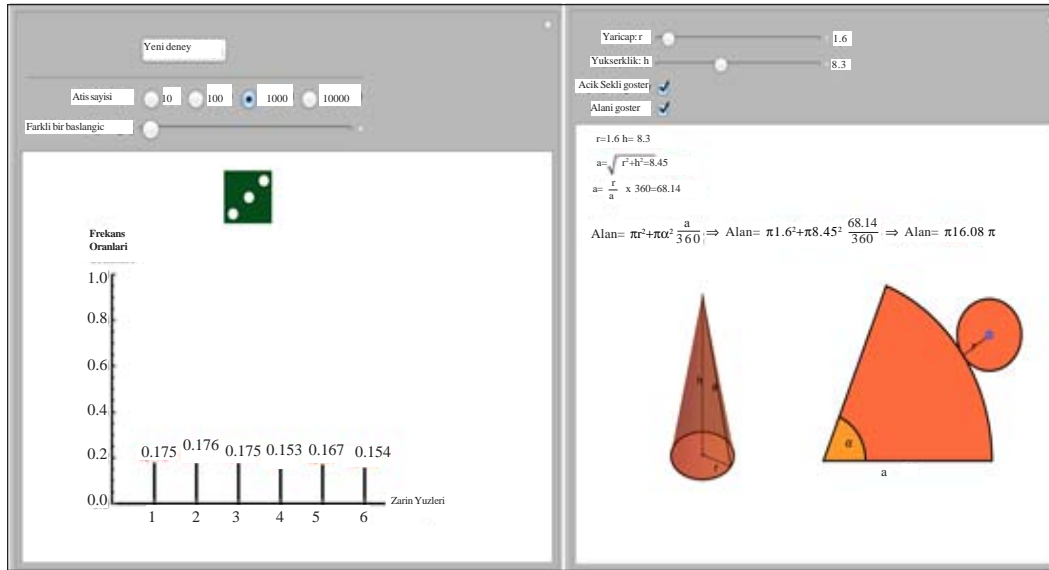
A pre and post-test experimental design was used to measure the efficiency of the method.

The participants were 146 middle school students in 8th grade from three private schools in Istanbul, of which the students have the same socioeconomic status. 67 of them were male and 79 were female. Two classes of each school were randomly assigned as the treatment and the control groups. In each school one instructor taught both classes to reduce the teacher effects. The study was conducted within a period of 14 weeks in the first term of the academic year of 2014-2015. In the treatment groups, the teachers extensively used the virtual manipulatives with an interactive whiteboard for all lessons while the traditional instruction was given to the control groups.

Prior to the first week of the study all of the students completed a teacher-created pretest, which contains 40 multiple-choice questions of subjects pertaining to the 7th grade curriculum to measure whether all the groups have the same mathematics achievement level on the average before the treatment. A similar test was applied at the end of the period to examine whether there were significant effects on mathematics achievement. ATMI was also carried out to all of the groups before and after the treatment to examine if the intensive and broad use of virtual manip-

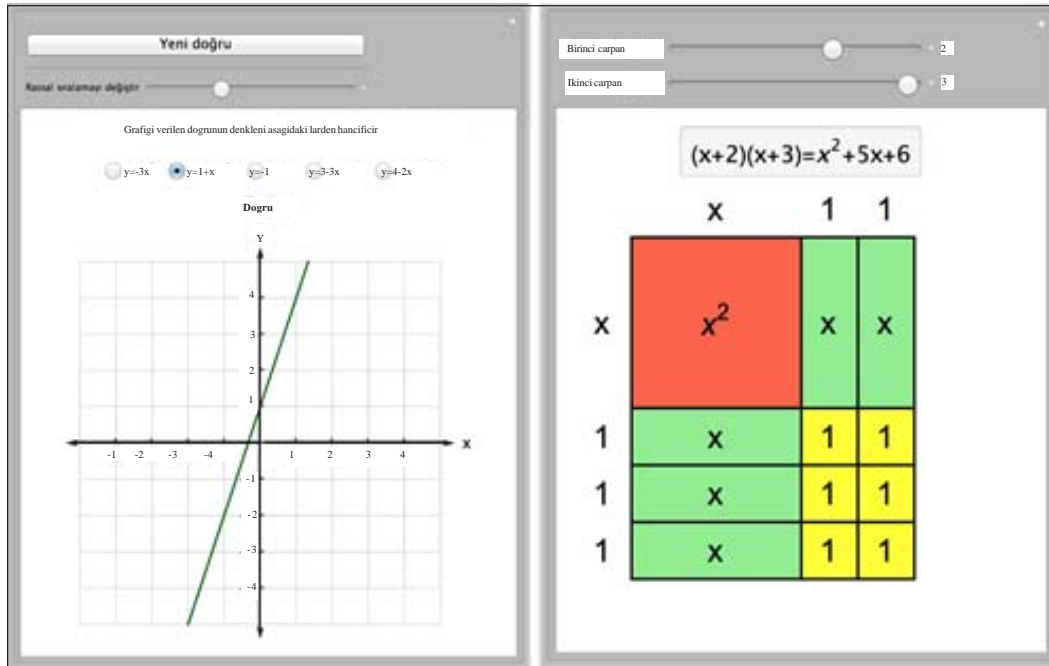


(a) Rotation



(b) Die simulation

(c) Cone



(d) Multiple choice test for equation of line

(e) Geometry of the product of two algebraic term

Fig. 1. (a) Rotation, (b) Die simulation, (c) Cone, (d) Multiple choice test for equation of line, (e) Geometry of the product of two algebraic term

Source: Author

ulatives during the lessons made a significant difference in attitude toward mathematics. The statistical findings obtained are presented in the next section.

RESULTS AND DISCUSSION

In this part of the paper, the researchers present the results dealing with the efficiency of the method on mathematics achievement and attitude, beginning with the findings on math achievement.

As mentioned above, the first intention was to measure if the groups have the same mathematics achievement level before the treatment. To do that the researchers carried out an exam with 40 multiple-choice questions of subjects pertaining to the 7th grade curriculum prior to the first week of the study. The grades of the achievement test were evaluated on a 0-100 scale. The reliability of the achievement exam was tested, and the Kuder-Richardson 20 reliability coefficient of it was found to be .83.

Table 1 shows the results of t-test and ANOVA dealing with the pre-test scores of all groups (the researchers preferred t-test instead of Mann-Whitney U test and ANOVA instead of Kruskal-Wallis test since the normality assumption is met for all). The following are the inferences based on these results:

- There is no significant difference between the means of the control groups ($p=0.240$)
- There is no significant difference between the means of the treatment groups ($p=0.103$)
- There is no significant difference between the means of treatment and control group of School 1 ($p=0.150$)
- There is no significant difference between the means of treatment and control group of School 2 ($p=0.110$)
- There is no significant difference between the means of treatment and control group of School 3 ($p=0.183$)

In short, the researchers can conclude that before the treatment that all the groups have the same mathematics achievement level on average.

At the end of 14 weeks, after the treatment, students completed 40 multiple-choice questions on subjects done in the first term of the 8th grade curriculum. Table 2 shows the results of paired t-test of pre and post-results of math achievement tests (the researchers preferred t-test instead of Wilcoxon Signed Rank test since the normality assumption is met). Since all the groups have the same mathematics achievement level before the treatment, the researchers combined the scores of three schools. The following are the inferences based on these results:

Table 2: The comparisons of pre-post test results of mathematics achievements

Groups	Pre-post	Mean	Std. deviation	Paired samples t-tests
Controls	Pre	63.62	10.44	$p=0.151$
	Post	65.99	10.11	
Treatments	Pre	65.36	9.64	$p=0.000$
	Post	74.08	9.01	

- There is no significant difference between the means of pre- and post scores of mathematics achievement tests for the control group ($p=0.151$).
- There is a highly significant difference between the means of pre and post-scores of mathematics achievement tests for the treatment group ($p=0.000$).

Therefore, it can be concluded that an intensive and broad use of virtual manipulatives during the lessons would make a significant difference in mathematics achievement.

As to examining the effects on attitude toward mathematics, ATMI was implemented to all the groups before the treatment. The researchers firstly carried out exploratory factor analysis to the data. The value of Cronbach's

Table 1: The comparisons of pre-test results of mathematics achievements

Pre-tests	Groups	Mean	Std. deviation	t-tests	ANOVA for treatments	ANOVA for controls
School 1	Control	63.82	11.39	$p=0.150$	$p=0.103$	$p=0.240$
	Treatment	68.17	8.36			
School 2	Control	61.00	8.65	$p=0.110$	$p=0.103$	$p=0.240$
	Treatment	65.71	11.25			
School 3	Control	66.04	11.30	$p=0.183$	$p=0.103$	$p=0.240$
	Treatment	62.19	9.32			

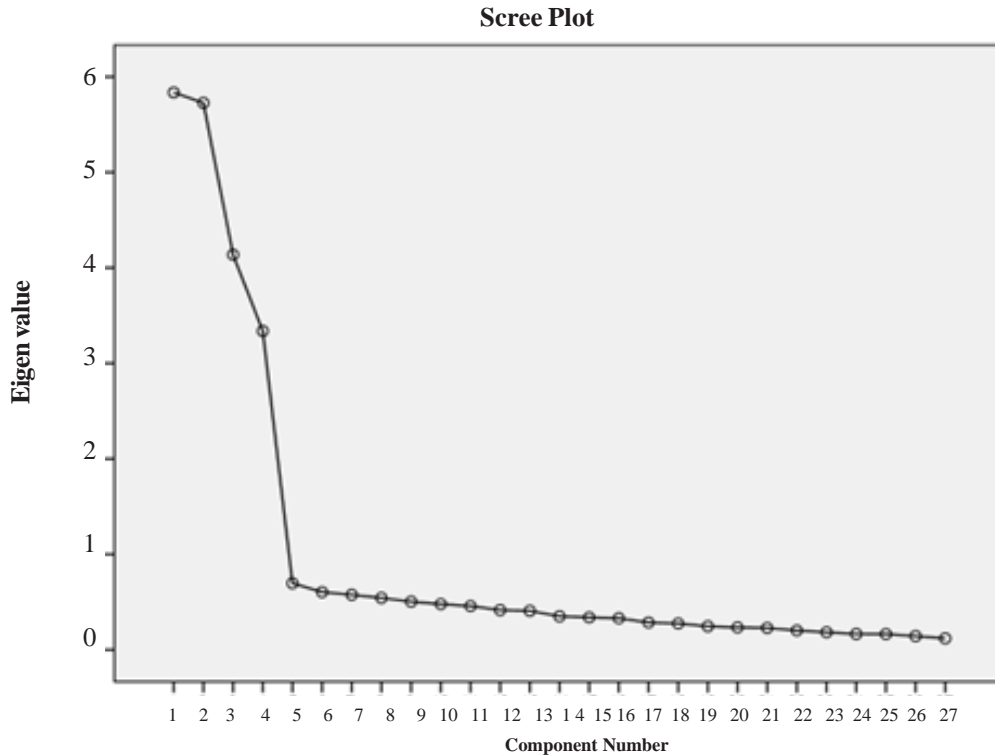


Fig. 2. Scree plot of attitude scores with 27 - item ATMI scale
Source: Author

alpha was .79 for the 40 items, showing an acceptable internal consistency. An item deletion process was performed in order to increase the value of alpha and to get a clearer picture of the factor analytic structure. Based on the criterion of retaining factors with eigenvalues greater than 1 and Scree test (see Fig. 2), the researchers reduced the 40 items to 27 with an alpha of .85 and four factors, which accounted for 70.5 percent of the total variance. As mentioned earlier, the four factors or dimensions of ATMI scale are

self-confidence, value, enjoyment and motivation. The item numbers of these four factors are 8, 5, 6 and 8, respectively. Since ATMI is a 5-point Likert scale, maximum scores of the factors are 40, 25, 30 and 40, respectively. ATMI was carried out after the treatment to examine the effects of virtual manipulatives on these factors separately. Out of the 40 items of ATMI, only the 27 reduced ones were used at this step.

Table 3 shows the results of paired t-test of pre- and post results of mathematics attitude

Table 3: The comparisons of pre-post test results of mathematics toward attitudes

Factors	Pre				Post				Paired sample T-tests	
	Control		Treatment		Control		Treatment		Cont.	Treat.
	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.		
Value	16.38	4.43	16.25	4.62	16.41	4.42	16.29	4.63	p=0.159	p=0.083
Self-confidence	22.73	6.55	26.07	6.37	22.74	6.60	30.14	6.40	p=0.567	p=0.000
Enjoyment	18.37	5.10	18.96	4.44	18.38	5.09	22.16	4.55	p=0.321	p=0.000
Motivation	23.78	6.52	23.12	6.65	23.71	6.49	28.31	6.71	p=0.582	p=0.000

scores (the researchers preferred t-test instead of Wilcoxon Signed Rank test since the normality assumption is met). The researchers combined the scores of three schools. The following are the inferences based on these results:

- For the control group, there is no significant difference between the means of pre- and post values of mathematics attitude for all factor levels.
- For the treatment group, there is a highly significant difference between the means of pre- and post values of mathematics attitude for all factor levels ($p=0.000$) except the factor, *Value*.

Therefore, it can be concluded that an intensive and broad use of virtual manipulatives during the lessons would make a significant difference in the attitude toward mathematics.

CONCLUSION

In this study, the researchers investigated the effects of an intensive and broad use of virtual manipulatives on the mathematics achievement and the attitudes of 8th grade students. The results revealed that using virtual manipulatives could offer an opportunity to improve the students' mathematics achievement and attitudes. Further studies should be carried out with different grade levels and different designs in order to confirm the actual benefits of virtual manipulatives.

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